

9 Work and Kinetic Energy



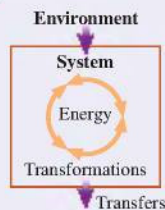
The bow may be very contemporary, but it's still the bow string doing work on the arrow that makes the arrow fly.

IN THIS CHAPTER, you will begin your study of how energy is transferred and transformed.

How should we think about energy?

Chapters 9 and 10 will develop the **basic energy model**, a powerful set of ideas for using energy. A key distinction is between the **system**, which has energy, and the **environment**. Energy can be **transferred** between the system and the environment or **transformed** within the system.

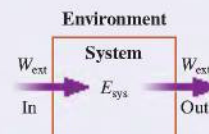
◀ LOOKING BACK Section 7.1 Interacting objects



What laws govern energy?

Working with energy is very much like accounting: A system's energy E changes by the amount of work done on the system. The mathematical statement of this idea is called the **energy principle**:

$$\Delta E_{\text{sys}} = W_{\text{ext}}$$

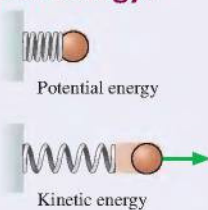


What are some important forms of energy?

Three important forms of energy:

- **Potential energy** is energy associated with an object's *position*.
- **Kinetic energy** is energy associated with an object's *motion*.
- **Thermal energy** is the energy of the random motion of *atoms* within an object.

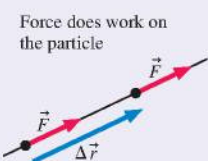
Energy is measured in **joules**.



What is work?

A process that **changes the energy of a system by mechanical means**—pushing or pulling on it—is called **work**.

Work W is done when a force pushes or pulls a particle through a displacement, thus changing the particle's kinetic energy.



What is power?

Power is the rate at which energy is transferred or transformed. For machines, power is the rate at which they do work. For electricity, power is the rate at which electric energy is transformed into heat, sound, or light. Power is measured in **watts**, where 1 watt is a rate of 1 joule per second.



Why is energy important?

Energy is one of the most important concepts in science, engineering, and society. Some would say it is *the* most important. All life depends on energy, transformed from solar energy to chemical energy to us. Society depends on energy, from industry and transportation to heating and cooling our buildings. Using energy wisely and efficiently is a key concern of the 21st century.

9.1 Energy Overview

Energy. It's a word you hear all the time, and everyone has some sense of what *energy* means. Moving objects have energy; energy is the ability to make things happen; energy is associated with heat and with electricity; we're constantly told to conserve energy; living organisms need energy; and engineers harness energy to do useful things. Some scientists consider the *law of conservation of energy* to be the most important of all the laws of nature. But all that in due time—first we have to start with the basic ideas.

Just what is energy? The concept of energy has grown and changed with time, and it is not easy to define in a general way just what energy is. Rather than starting with a formal definition, we're going to let the concept of energy expand slowly over the course of several chapters. Our goal is to understand the characteristics of energy, how energy is used, and how energy is transformed from one form into another. It's a complex story, so we'll take it step by step until all the pieces are in place.

Some important forms of energy

Kinetic energy K



Kinetic energy is the energy of motion. All moving objects have kinetic energy. The more massive an object or the faster it moves, the larger its kinetic energy.

Potential energy U



Potential energy is stored energy associated with an object's position. The roller coaster's gravitational potential energy depends on its height above the ground.

Thermal energy E_{th}



Thermal energy is the sum of the microscopic kinetic and potential energies of all the atoms and bonds that make up the object. An object has more thermal energy when hot than when cold.

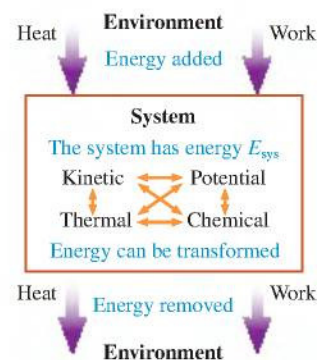
The Energy Principle

« Section 7.1 introduced *interaction diagrams* and the very important distinction between the **system**, those objects whose motion and interactions we wish to analyze, and the **environment**, objects external to the system but exerting forces on the system. The most important step in an energy analysis is to clearly define the system. Why? Because energy is not some disembodied, ethereal substance; it's the energy of *something*. Specifically, it's *the energy of a system*.

FIGURE 9.1 illustrates the idea pictorially. The system *has energy*, the **system energy**, which we'll designate E_{sys} . There are many kinds or forms of energy: kinetic energy K , potential energy U , thermal energy E_{th} , chemical energy, and so on. We'll introduce these one by one as we go along. Within the system, energy can be *transformed without loss*. Chemical energy can be transformed into kinetic energy, which is then transformed into thermal energy. As long as the system is not interacting with the environment, the total energy of the system is unchanged. You'll recognize this idea as an initial statement of the *law of conservation of energy*.

But systems often do interact with their environment. Those interactions *change* the energy of the system, either increasing it (energy added) or decreasing it (energy removed). We say that interactions with the environment *transfer* energy into or out of the system. Interestingly, there are only two ways to transfer energy. One is by mechanical means, using forces to push and pull on the system. A process that

FIGURE 9.1 A system-environment perspective on energy.



transfers energy to or from a system by mechanical means is called **work**, with the symbol W . We'll have a lot to say about work in this chapter. The second is by thermal means when the environment is hotter or colder than the system. A process that transfers energy to or from a system by thermal means is called **heat**. We'll defer a discussion of heat until Chapter 18, but we wanted to mention it now in order to gain an overview of what the energy story is all about.

Some energy transfers ...

**Putting a shot****System:** The shot**Transfer:** $W \rightarrow K$

The athlete (the environment) does work pushing the shot to give it kinetic energy.

**Pulling a slingshot****System:** The slingshot**Transfer:** $W \rightarrow U$

The boy (the environment) does work by stretching the rubber band to give it potential energy.

A falling diver**System:** The diver and the earth**Transformation:** $U \rightarrow K$

The diver is speeding up as gravitational potential energy is transformed into kinetic energy.

**A speeding meteor****System:** The meteor and the air**Transformation:** $K \rightarrow E_{\text{th}}$

The meteor and the air get hot enough to glow as the meteor's kinetic energy is transformed into thermal energy.



... and transformations

The key ideas are **energy transfer** between the environment and the system and **energy transformation** within the system. This is much like what happens with money. You may have several accounts at the bank—perhaps a checking account and a couple of savings accounts. You can move money back and forth between the accounts, thus transforming it without changing the total amount of money. Of course, you can also transfer money into or out of your accounts by making deposits or withdrawals. If we treat a withdrawal as a negative deposit—which is exactly what accountants do—simple accounting tells you that

$$\Delta(\text{balance}) = \text{net deposit}$$

That is, the change in your bank balance is simply the sum of all your deposits.

Energy accounting works the same way. Transformations of energy within the system move the energy around but don't change the total energy of the system. *Change* occurs only when there's a transfer of energy between the system and the environment. If we treat incoming energy as a positive transfer and outgoing energy as a negative transfer, and with work being the only energy-transfer process that we consider for now, we can write

$$\Delta E_{\text{sys}} = W_{\text{ext}} \quad (9.1)$$

where the subscript on W refers to external work done by the environment. This very simple looking statement, which is just a statement of energy accounting, is called the **energy principle**. But don't let the simplicity fool you; this will turn out to be an incredibly powerful tool for analyzing physical situations and solving problems.

The Basic Energy Model

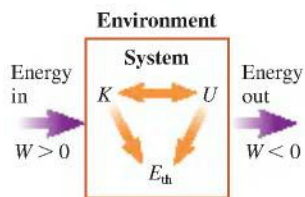
We'll complete our energy overview—a roadmap of the next two chapters—with the **basic energy model**.

MODEL 9.1

Basic energy model

Energy is a property of the system.

- Energy is *transformed* within the system without loss.
- Energy is *transferred* to and from the system by forces from the environment.
 - The forces do *work* on the system.
 - $W > 0$ for energy added.
 - $W < 0$ for energy removed.
- The energy of an *isolated system*—one that doesn't interact with its environment—does not change. We say it is *conserved*.
- The energy principle is $\Delta E_{\text{sys}} = W_{\text{ext}}$.
- Limitations: Model fails if there is energy transfer via thermal processes (heat).



Exercise 1

We call this model *basic* because, for now, the only forms of energy we'll consider are kinetic energy, potential energy, and thermal energy, and the only energy-transfer process we'll consider is work. This is an excellent model for a mechanical process, but it's not complete. We'll expand the model when we get to thermodynamics by adding chemical energy, another form of energy, and heat, another energy-transfer process. And this model, although basic, still has many complexities, so we'll be developing it piece by piece in this chapter and the next.

9.2 Work and Kinetic Energy for a Single Particle

Let's start our investigation of energy with the simplest possible situation: One particle of mass m is acted on by one constant force \vec{F} that acts parallel to the direction of motion, pushing or pulling on the particle as it undergoes a displacement Δs . We define the particle to be the system—a one-particle system—while the agent of the force is in the environment.

FIGURE 9.2 shows both an interaction diagram and a new kind of pictorial representation, a **before-and-after representation**, in which we show an object *before* and *after* an interaction and, as usual, establish a coordinate system and define appropriate symbols.

You know what's going to happen. If the force is in the direction of motion—the situation shown in the figure—the particle will speed up and its “energy of motion” will increase. Conversely, if the force opposes the motion, the particle will slow down and lose energy. Our goal is to make this idea precise by discovering exactly how the changing energy is related to the applied force.

We'll start by writing Newton's second law for the particle:

$$F_s = ma_s = m \frac{dv_s}{dt} \quad (9.2)$$

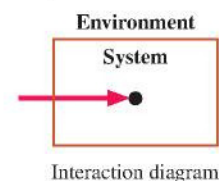
Newton's second law tells us how the particle's velocity changes with time. But suppose we want to know how the velocity changes with position. To answer that question, we can use the chain rule that you've learned in calculus:

$$\frac{dv_s}{dt} = \frac{dv_s}{ds} \frac{ds}{dt} = v_s \frac{dv_s}{ds} \quad (9.3)$$

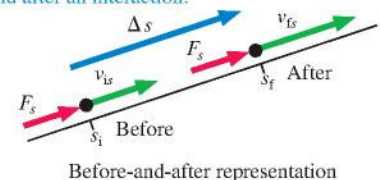
where in the last step we used $ds/dt = v_s$. With this, we can write Newton's second law as

$$F_s = mv_s \frac{dv_s}{ds} \quad (9.4)$$

FIGURE 9.2 The interaction diagram and before-and-after representation for a one-particle system.



A before-and-after representation shows the object's position and velocity before and after an interaction.



Now we have an alternative version of Newton's second law in terms of the rate of change of velocity with position.

To use Equation 9.4, we first rewrite it as

$$mv_s dv_s = F_s ds \quad (9.5)$$

Now we can integrate. This is going to be a definite integral over just the motion shown in the before-and-after representation. That is, the right side will be an integral over position s from the initial position s_i to the final position s_f . The left side will be an integral over velocity v_s , and its limits have to match the limits of the right-hand integral: from v_{is} at s_i to v_{fs} at s_f . Thus we have

$$\int_{v_{is}}^{v_{fs}} mv_s dv_s = \int_{s_i}^{s_f} F_s ds \quad (9.6)$$

We have two integrals to examine, and we'll do them one by one. We can start with the integral on the left, which is of the form $\int x dx$. Factoring out m , which is a constant, we find

$$m \int_{v_{is}}^{v_{fs}} v_s dv_s = m \left[\frac{1}{2} v_s^2 \right]_{v_{is}}^{v_{fs}} = \frac{1}{2} m v_{fs}^2 - \frac{1}{2} m v_{is}^2 = \Delta \left(\frac{1}{2} m v^2 \right) = \Delta K \quad (9.7)$$

You'll notice that we dropped the subscript s in the next-to-last step. v_s is a vector component, with a sign to indicate direction, but the sign makes no difference after v_s is squared. All that matters is the particle's *speed* v .

The last step in Equation 9.7 introduces a new quantity

$$K = \frac{1}{2} m v^2 \quad (\text{kinetic energy}) \quad (9.8)$$

which is called the **kinetic energy** of the particle. **Kinetic energy is energy of motion.** It depends on the particle's mass and speed but not on its position. Furthermore, kinetic energy is a property or characteristic of the system. So what we've calculated with the left-hand integral is $\Delta K = K_f - K_i$, the *change* in the system's kinetic energy as the force pushes the particle through the displacement Δs . ΔK is positive if the particle speeds up (gain of kinetic energy), negative if it slows down (loss of kinetic energy).

NOTE By its definition, kinetic energy can *never* be negative. Finding a negative value for K while solving a problem is an indication that you've made a mistake somewhere.

The unit of kinetic energy is mass multiplied by velocity squared. In SI units, this is $\text{kg m}^2/\text{s}^2$. Because energy is so important, the unit of energy is given its own name, the **joule**. We define

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2$$

All other forms of energy are also measured in joules.

To give you an idea about the size of a joule, consider a 0.5 kg mass (≈ 1 lb on earth) moving at 4 m/s (≈ 10 mph). Its kinetic energy is

$$K = \frac{1}{2} m v^2 = \frac{1}{2} (0.5 \text{ kg}) (4 \text{ m/s})^2 = 4 \text{ J}$$

This suggests that everyday objects moving at ordinary speeds will have energies from a fraction of a joule up to, perhaps, a few thousand joules. A running person has $K \approx 1000 \text{ J}$, while a high-speed truck might have $K \approx 10^6 \text{ J}$.

NOTE You *must* have masses in kilograms and velocities in m/s before doing energy calculations.

STOP TO THINK 9.1 A 1000 kg car has a speed of 20 m/s. A 2000 kg truck has a speed of 10 m/s. Which has more kinetic energy?

- a. The car. b. The truck. c. Their kinetic energies are the same.

Work

Now let's turn to the integral on the right-hand side of Equation 9.6. This integral is telling us *by how much* the kinetic energy changes due to the force. That is, it is the energy transferred to or from the system by the force. Earlier we said that a process that transfers energy to or from a system by mechanical means—by forces—is called work. So the integral on the right-hand side of Equation 9.6 must be the *work W done by force \vec{F}* .

Having identified the left side of Equation 9.6 with the changing kinetic energy of the system and the right side with the work done on the system, we can rewrite Equation 9.6 as

$$\Delta K = K_f - K_i = W \quad (9.9)$$

(Energy principle for a one-particle system)

Change in the system's kinetic energy Amount of work done by an external force
Final kinetic energy Initial kinetic energy

This is our first version of the energy principle. Notice that it's a cause-and-effect statement: **The work done on a one-particle system causes the system's kinetic energy to change.**

We'll study work thoroughly in the next section, but for now we're considering only the simplest case of a constant force parallel to the direction of motion (the s -axis). A constant force can be factored out of the integral, giving

$$W = \int_{s_i}^{s_f} F_s ds = F_s \int_{s_i}^{s_f} ds = F_s s \Big|_{s_i}^{s_f} = F_s (s_f - s_i) = F_s \Delta s \quad (9.10)$$

The unit of work, that of force multiplied by distance, is the Nm. Recall that $1 \text{ N} = 1 \text{ kg m/s}^2$. Thus

$$1 \text{ Nm} = 1 (\text{kg m/s}^2) \text{ m} = 1 \text{ kg m}^2/\text{s}^2 = 1 \text{ J}$$

Thus the unit of work is really the unit of energy. This is consistent with the idea that work is a transfer of energy. Rather than use Nm, we will measure work in joules.

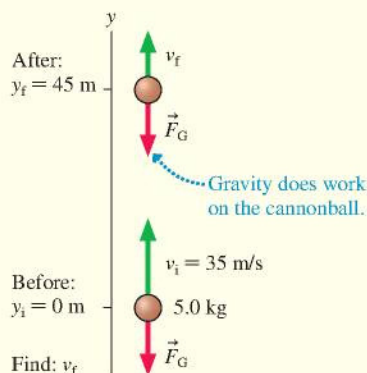
EXAMPLE 9.1 Firing a cannonball

A 5.0 kg cannonball is fired straight up at 35 m/s. What is its speed after rising 45 m?

MODEL Let the system consist of only the cannonball, which we model as a particle. Assume that air resistance is negligible.

VISUALIZE FIGURE 9.3 is a before-and-after pictorial representation. Because before-and-after representations are usually simpler than

FIGURE 9.3 Before-and-after representation of the cannonball.



the pictorial representation used in dynamics problems, you can include known information right on the diagram instead of making a Known table.

SOLVE It isn't necessary to use work and energy to solve this problem. You could solve it as a free-fall problem. Or you might have previously learned to solve problems like this using potential energy, a topic we'll take up in the next chapter. But using work and energy emphasizes how these two key ideas are related, and it gives us a simple example of the *problem-solving process* before we get to more complex problems. The energy principle is $\Delta K = W$, where work is done by the force of gravity. The cannonball is rising, so its displacement Δy is positive. But the force vector points down, with component $F_y = -mg$. Thus gravity does work

$$W = F_y \Delta y = -mg \Delta y = -(5.0 \text{ kg})(9.80 \text{ m/s}^2)(45 \text{ m}) = -2210 \text{ J}$$

as the cannonball rises 45 m. A negative work means that the system is losing energy, which is what we expect as the cannonball slows.

Continued

The cannonball's change of kinetic energy is $\Delta K = K_f - K_i$.
The initial kinetic energy is

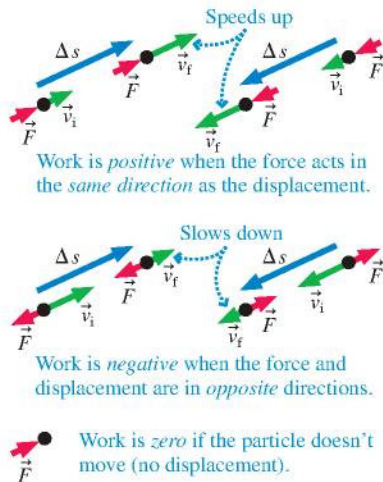
$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(5.0 \text{ kg})(35 \text{ m/s})^2 = 3060 \text{ J}$$

Using the energy principle, we find the final kinetic energy to be $K_f = K_i + W = 3060 \text{ J} - 2210 \text{ J} = 850 \text{ J}$. Then

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(850 \text{ J})}{5.0 \text{ kg}}} = 18 \text{ m/s}$$

ASSESS 35 m/s \approx 70 mph. A cannonball fired upward at that speed is going to go fairly high. To have lost half its speed at a height of 45 m \approx 150 ft seems reasonable.

FIGURE 9.4 How to determine the sign of W .



Signs of Work

Work can be either positive or negative, but some care is needed to get the sign right when calculating work. The key is to remember that work is an energy transfer. If the force causes the particle to speed up, then the work done by that force is positive. Similarly, negative work means that the force is causing the object to slow and lose energy.

The sign of W is *not* determined by the direction the force vector points. That's only half the issue. The displacement Δs also has a sign, so you have to consider both the force direction *and* the displacement direction. As **FIGURE 9.4** shows, **work is positive when the force acts in the direction of the displacement** (causing the particle to speed up). Similarly, **work is negative when force and displacement are in opposite directions** (causing the particle to slow). And there's no work at all ($W = 0$) if the particle doesn't move!

STOP TO THINK 9.2 A rock falls to the bottom of a deep canyon. Is the work done on the rock by gravity positive, negative, or zero?

Extending the Model

Our initial model has been of a single particle acted on by a constant force parallel to the displacement. We can easily make some straightforward extensions of this model to slightly more complex—and interesting—situations:

- **Force perpendicular to the displacement:** A force parallel to a particle's displacement causes the particle to speed up or slow down, changing its energy. But a force *perpendicular* to the displacement does *not* change the particle's speed; it is neither speeding up nor slowing down. Its energy is not changing, so no work is being done on it. **A force perpendicular to the displacement does no work.**
- **Multiple forces:** If multiple forces act on a system, their works add. That is, $\Delta K = W_{\text{tot}}$, where the total work done is

$$W_{\text{tot}} = W_1 + W_2 + W_3 + \dots \quad (9.11)$$

- **Multiparticle systems:** If a system has more than one particle, the system's energy is the total kinetic energy of all the particles:

$$E_{\text{sys}} = K_{\text{tot}} = K_1 + K_2 + K_3 + \dots \quad (9.12)$$

K_{tot} is truly a *system* energy, not the energy of any one particle. How does K_{tot} change when work is done? You can see from its definition that ΔK_{tot} is the sum of all the individual kinetic-energy changes, and each of those changes is the work done on that particular particle. Thus

$$\Delta K_{\text{tot}} = W_{\text{tot}} \quad (9.13)$$

where now W_{tot} is the total work done on *all* the particles in the system.

NOTE You might expect $W_{\text{tot}} = (F_{\text{net}})_s \Delta s$, where \vec{F}_{net} is the net work on the system. This is true for a one-particle system (if all the forces are constant), but in general it is *not* true for a multiparticle system because each particle undergoes a different displacement. You must find the work done on each particle, then sum those to find the total work done on the system.

STOP TO THINK 9.3 Two equal-mass pucks on frictionless ice are pushed toward each other by two equal but opposite forces. Is the total work positive, negative, or zero?



9.3 Calculating the Work Done

Section 9.2 introduced two key ideas: (1) a system has energy and (2) work is a mechanical process that changes the system's energy. Now we're ready to look more closely at how to calculate the work done in different situations. Although we'll be focusing on the mathematical techniques of calculating work, it's important to keep in mind that our real goal is to learn how the energy of a system changes when forces are applied to it.

“Work” is a common word in the English language, with many meanings. Work might refer to physical exertion, to your job or occupation, or even to a work of art. But set aside those ideas about work because they are *not* what work means in physics. Work, as we'll use the word, is a *process*. Specifically, it is a process that changes a system's energy by mechanical means—pushing or pulling on it with forces. We say that work *transfers* energy between the environment and the system.

Equation 9.6 defined work as

$$W = \int_{s_i}^{s_f} F_s ds \quad (9.14)$$

(work done by force \vec{F} as a particle is displaced from s_i to s_f)

where, to remind you, F_s is the component of \vec{F} in the direction of motion (the s -direction). We began by looking at a force that was parallel to the displacement, but such a restriction is not required because any force component perpendicular to the motion does no work. Equation 9.14 is, in fact, a general definition of work.

We'll start by learning how to calculate work for constant forces, and we'll introduce a new mathematical idea, the *dot product* of two vectors, that will allow us to write the work in a compact notation. Then we'll consider the work done by a variable force that changes as the particle moves.

Constant Force

FIGURE 9.5 shows a particle moving in a straight line. A constant force \vec{F} , which makes an angle θ with respect to the particle's displacement $\Delta\vec{r}$, acts on the particle throughout its motion. We've established an s -axis in the direction of motion, and you can see that the force component along the direction of motion is $F_s = F \cos \theta$. According to Equation 9.14, the work done on the particle by this force is

$$W = \int_{s_i}^{s_f} F_s ds = \int_{s_i}^{s_f} F \cos \theta ds \quad (9.15)$$

Both F and $\cos \theta$ are constant, so they can be taken outside the integral. Thus

$$W = F \cos \theta \int_{s_i}^{s_f} ds = F \cos \theta (s_f - s_i) \quad (9.16)$$

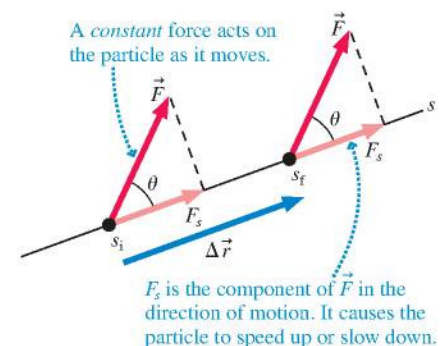
Now s_i and s_f are specific to this coordinate system, but their *difference* $s_f - s_i$ is Δr , the magnitude of the particle's displacement vector. Thus we can write more generally, independent of any specific coordinate system, that the work done by the constant force \vec{F} is

$$W = F(\Delta r) \cos \theta \quad (\text{work done by a constant force}) \quad (9.17)$$

where θ is the angle between the force and the particle's displacement $\Delta\vec{r}$.

NOTE You may have learned in an earlier physics course that work is “force times distance.” This is *not* the definition of work, merely a special case. Work is “force times distance” only if the force is constant *and* parallel to the displacement ($\theta = 0^\circ$).

FIGURE 9.5 Work being done by a constant force.



EXAMPLE 9.2 Pulling a suitcase

A strap inclined upward at a 45° angle pulls a suitcase 100 m through the airport. The tension in the strap is 20 N. How much work does the tension force do on the suitcase?

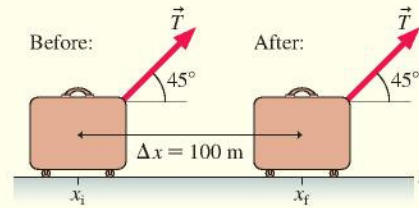
MODEL Let the system consist of only the suitcase, which we model as a particle.

VISUALIZE FIGURE 9.6 is a before-and-after pictorial representation.

SOLVE The motion is along the x -axis, so in this case $\Delta r = \Delta x$. We can use Equation 9.17 to find that the tension does work:

$$W = T(\Delta x) \cos \theta = (20 \text{ N})(100 \text{ m}) \cos 45^\circ = 1400 \text{ J}$$

FIGURE 9.6 Pictorial representation of the suitcase.



ASSESS Because a person pulls the strap, we would say informally that the person does 1400 J of work on the suitcase.

According to the basic energy model, work can be either positive or negative to indicate energy transfer into or out of the system. The quantities F and Δr are always positive, so the sign of W is determined entirely by the angle θ between the force \vec{F} and the displacement $\Delta \vec{r}$.

TACTICS BOX 9.1**Calculating the work done by a constant force**

Force and displacement	θ	Work W	Sign of W	Energy transfer
	0°	$F(\Delta r)$	+	Energy is transferred into the system. The particle speeds up. K increases.
	$< 90^\circ$	$F(\Delta r) \cos \theta$	+	
	90°	0	0	No energy is transferred. Speed and K are constant.
	$> 90^\circ$	$F(\Delta r) \cos \theta$	-	Energy is transferred out of the system. The particle slows down. K decreases.
	180°	$-F(\Delta r)$	-	

Exercises 3–9



NOTE The sign of W depends on the angle between the force vector and the displacement vector, *not* on the coordinate axes. A force to the left does *positive* work if it pushes a particle to the left (the force and the displacement are in the same direction) even though the force component F_x is negative. Think about whether the force is trying to increase the particle's speed ($W > 0$) or decrease the particle's speed ($W < 0$).

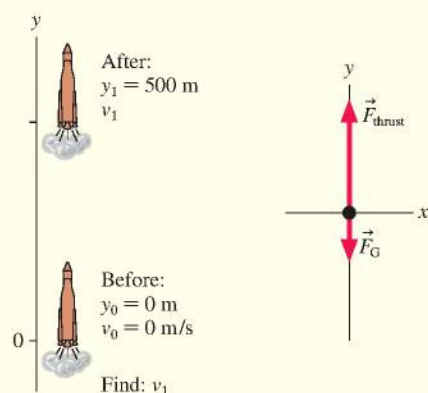
EXAMPLE 9.3 Launching a rocket

A 150,000 kg rocket is launched straight up. The rocket motor generates a thrust of 4.0×10^6 N. What is the rocket's speed at a height of 500 m?

MODEL Let the system consist of only the rocket, which we model as a particle. Thrust and gravity are constant forces that do work on the rocket. We'll ignore air resistance and any slight mass loss.

VISUALIZE FIGURE 9.7 shows a before-and-after representation and a free-body diagram.

FIGURE 9.7 Before-and-after representation and free-body diagram of a rocket launch.



SOLVE We can solve this problem with the energy principle, $\Delta K = W_{\text{tot}}$. Both forces do work on the rocket. The thrust is in the direction of motion, with $\theta = 0^\circ$, and thus

$$W_{\text{thrust}} = F_{\text{thrust}}(\Delta r) = (4.0 \times 10^6 \text{ N})(500 \text{ m}) = 2.00 \times 10^9 \text{ J}$$

The gravitational force points downward, opposite the displacement $\Delta \vec{r}$, so $\theta = 180^\circ$. Thus the work done by gravity is

$$W_{\text{grav}} = -F_G(\Delta r) = -mg(\Delta r) \\ = -(1.5 \times 10^5 \text{ kg})(9.8 \text{ m/s}^2)(500 \text{ m}) = -0.74 \times 10^9 \text{ J}$$

The work done by the thrust is positive. By itself, the thrust would cause the rocket to speed up. The work done by gravity is negative, not because \vec{F}_G points down but because \vec{F}_G is opposite the displacement. By itself, gravity would cause the rocket to slow down. The energy principle, using $K_i = 0$, is

$$\Delta K = \frac{1}{2}mv_1^2 - 0 = W_{\text{tot}} = W_{\text{thrust}} + W_{\text{grav}} = 1.26 \times 10^9 \text{ J}$$

Solving for the speed, we find

$$v_1 = \sqrt{\frac{2W_{\text{tot}}}{m}} = 130 \text{ m/s}$$

ASSESS The total work is positive, meaning that energy is transferred to the rocket. In response, the rocket speeds up.

STOP TO THINK 9.4 A crane uses a single cable to lower a steel girder into place. The girder moves with constant speed. The cable tension does work W_T and gravity does work W_G . Which statement is true?

- W_T is positive and W_G is positive.
- W_T is positive and W_G is negative.
- W_T is negative and W_G is positive.
- W_T is negative and W_G is negative.
- W_T and W_G are both zero.

Work as a Dot Product of Two Vectors

There's something different about the quantity $F(\Delta r)\cos\theta$ in Equation 9.17. We've spent many chapters adding vectors, but this is the first time we've *multiplied* two vectors. Multiplying vectors is not like multiplying scalars. In fact, there is more than one way to multiply vectors. We will introduce one way now, the *dot product*.

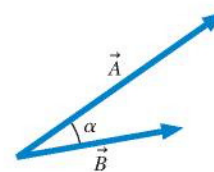
FIGURE 9.8 shows two vectors, \vec{A} and \vec{B} , with angle α between them. We define the **dot product** of \vec{A} and \vec{B} as

$$\vec{A} \cdot \vec{B} = AB\cos\alpha \quad (9.18)$$

A dot product *must have* the dot symbol \cdot between the vectors. The notation $\vec{A}\vec{B}$, without the dot, is *not* the same thing as $\vec{A} \cdot \vec{B}$. The dot product is also called the **scalar product** because the value is a scalar. Later, when we need it, we'll introduce a different way to multiply vectors called the *cross product*.

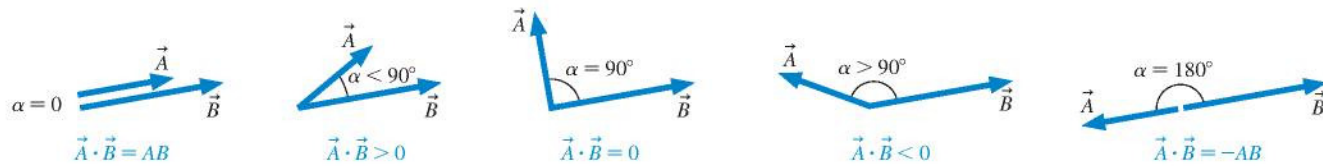
The dot product of two vectors depends on the orientation of the vectors. **FIGURE 9.9** shows five different situations, including the three "special cases" where $\alpha = 0^\circ$, 90° , and 180° .

FIGURE 9.8 Vectors \vec{A} and \vec{B} , with angle α between them.



NOTE The dot product of a vector with itself is well defined. If $\vec{B} = \vec{A}$ (i.e., \vec{B} is a copy of \vec{A}), then $\alpha = 0^\circ$. Thus $\vec{A} \cdot \vec{A} = A^2$.

FIGURE 9.9 The dot product $\vec{A} \cdot \vec{B}$ as α ranges from 0° to 180° .



EXAMPLE 9.4 Calculating a dot product

Compute the dot product of the two vectors in **FIGURE 9.10**.

SOLVE The angle between the vectors is $\alpha = 30^\circ$, so

$$\vec{A} \cdot \vec{B} = AB \cos \alpha = (3)(4) \cos 30^\circ = 10.4$$

FIGURE 9.10 Vectors \vec{A} and \vec{B} of Example 9.4.

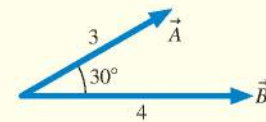
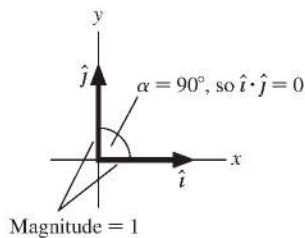


FIGURE 9.11 The unit vectors \hat{i} and \hat{j} .



Like vector addition and subtraction, calculating the dot product of two vectors is often performed most easily using vector components. **FIGURE 9.11** reminds you of the unit vectors \hat{i} and \hat{j} that point in the positive x -direction and positive y -direction. The two unit vectors are perpendicular to each other, so their dot product is $\hat{i} \cdot \hat{j} = 0$. Furthermore, because the magnitudes of \hat{i} and \hat{j} are 1, $\hat{i} \cdot \hat{i} = 1$ and $\hat{j} \cdot \hat{j} = 1$.

In terms of components, we can write the dot product of vectors \vec{A} and \vec{B} as

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j}) \cdot (B_x \hat{i} + B_y \hat{j})$$

Multiplying this out, and using the results for the dot products of the unit vectors:

$$\begin{aligned} \vec{A} \cdot \vec{B} &= A_x B_x \hat{i} \cdot \hat{i} + (A_x B_y + A_y B_x) \hat{i} \cdot \hat{j} + A_y B_y \hat{j} \cdot \hat{j} \\ &= A_x B_x + A_y B_y \end{aligned} \quad (9.19)$$

That is, the dot product is the sum of the products of the components.

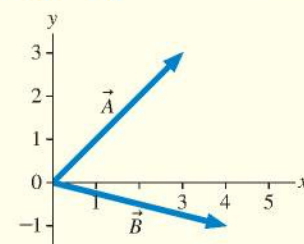
EXAMPLE 9.5 Calculating a dot product using components

Compute the dot product of $\vec{A} = 3\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} - \hat{j}$.

SOLVE **FIGURE 9.12** shows vectors \vec{A} and \vec{B} . We could calculate the dot product by first doing the geometry needed to find the angle between the vectors and then using Equation 9.18. But calculating the dot product from the vector components is much easier. It is

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (3)(4) + (3)(-1) = 9$$

FIGURE 9.12 Vectors \vec{A} and \vec{B} .



Looking at Equation 9.17, the work done by a constant force, you should recognize that it is the dot product of the force vector and the displacement vector:

$$W = \vec{F} \cdot \Delta \vec{r} \quad (\text{work done by a constant force}) \quad (9.20)$$

This definition of work is valid for a constant force.

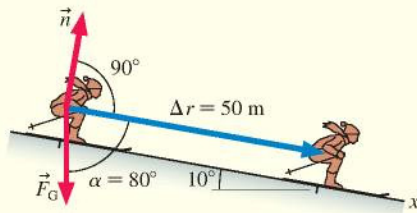
EXAMPLE 9.6 Calculating work using the dot product

A 70 kg skier is gliding at 2.0 m/s when he starts down a very slippery 50-m-long, 10° slope. What is his speed at the bottom?

MODEL Model the skier as a particle and interpret “very slippery” to mean frictionless. Use the energy principle to find his final speed.

VISUALIZE FIGURE 9.13 shows a pictorial representation.

FIGURE 9.13 Pictorial representation of the skier.



Before:

$$\begin{aligned}x_0 &= 0 \text{ m} \\v_0 &= 2.0 \text{ m/s} \\m &= 70 \text{ kg}\end{aligned}$$

After:

$$x_1 = 50 \text{ m}$$

$$v_1$$

Find: v_1

SOLVE The only forces on the skier are \vec{F}_G and \vec{n} . The normal force is perpendicular to the motion and thus does no work. The work done by gravity is easily calculated as a dot product:

$$\begin{aligned}W &= \vec{F}_G \cdot \Delta\vec{r} = mg(\Delta r)\cos\alpha \\ &= (70 \text{ kg})(9.8 \text{ m/s}^2)(50 \text{ m})\cos 80^\circ = 5960 \text{ J}\end{aligned}$$

Notice that the angle *between* the vectors is 80° , not 10° . Then, from the energy principle, we find

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = W \\v_1 &= \sqrt{v_0^2 + \frac{2W}{m}} = \sqrt{(2.0 \text{ m/s})^2 + \frac{2(5960 \text{ J})}{70 \text{ kg}}} = 13 \text{ m/s}\end{aligned}$$

NOTE While in the midst of the mathematics of calculating work, do not lose sight of what the energy principle is all about. It is a statement about *energy transfer*: Work causes a particle’s kinetic energy to either increase or decrease.

STOP TO THINK 9.5 Which force does the most work as a particle undergoes displacement $\Delta\vec{r}$?

- The 10 N force.
- The 8 N force.
- The 6 N force.
- They all do the same amount of work.

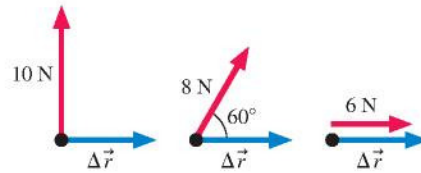
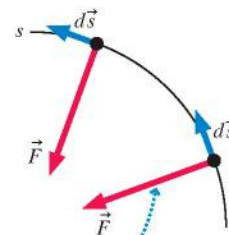


FIGURE 9.14 A perpendicular force does no work.



The force is everywhere perpendicular to the displacement, so it does no work.

Zero-Work Situations

There are three common situations where *no* work is done. The most obvious is when the object doesn’t move ($\Delta s = 0$). If you were to hold a 200 lb weight over your head, you might break out in a sweat and your arms would tire. You might feel that you had done a lot of work, but you would have done *zero* work in the physics sense because the weight was not displaced and thus you transferred no energy to it. **A force acting on a particle does no work unless the particle is displaced.**

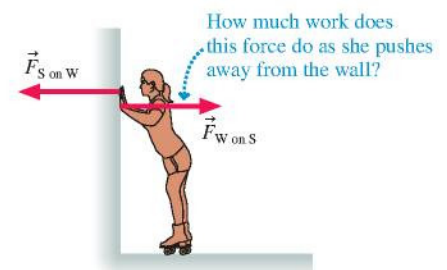
FIGURE 9.14 shows a particle moving in uniform circular motion. As you learned in Chapter 8, uniform circular motion requires a force pointing toward the center of the circle. How much work does this force do?

Zero! You can see that the force is everywhere perpendicular to the small displacement $d\vec{s}$, so the dot product is zero and the force does *no* work on the particle. This shouldn’t be surprising. The particle’s speed, and hence its kinetic energy, doesn’t change in uniform circular motion, so no energy is transferred to or from the system. **A force everywhere perpendicular to the motion does no work.** The friction force on a car turning a corner does no work. Neither does the tension force when a ball on a string is in circular motion.

Last, consider the roller skater in **FIGURE 9.15** who straightens her arms and pushes off from a wall. She applies a force to the wall and thus, by Newton’s third law, the wall applies a force $\vec{F}_{W \text{ on } S}$ to her. How much work does this force do?

Surprisingly, zero. The reason is subtle but worth discussing because it gives us insight into how energy is transferred and transformed. The skater differs from suitcases and rockets in two important ways. First, the skater, as she extends her arms,

FIGURE 9.15 Does the wall do work on the skater?



is a *deformable object*. We cannot use the particle model for a deformable object. Second, the skater has an *internal source of energy*. Because she's a living object, she has an internal store of chemical energy that is available through metabolic processes.

Although the skater's center of mass is displaced, *the palms of her hands—where the force is exerted—are not*. The particles on which force $F_{W \text{ on } S}$ acts have no displacement, and we've just seen that there's no work without displacement. The force acts, but the force doesn't push any physical thing through a displacement. Hence no work is done.

But the skater indisputably gains kinetic energy. How? Recall, from the energy overview that started this chapter, that the full energy principle is $\Delta E_{\text{sys}} = W_{\text{ext}}$. A system can gain kinetic energy without any work being done *if* it can transform some other energy into kinetic energy. In this case, the skater transforms chemical energy into kinetic energy. The same is true if you jump straight up from the ground. The ground applies an upward force to your feet, but that force does no work because the point of application—the soles of your feet—has no displacement while you're jumping. Instead, your increased kinetic energy comes via a decrease in your body's chemical energy. A brick cannot jump or push off from a wall because it cannot deform and has no usable source of internal energy.

STOP TO THINK 9.6 A car accelerates smoothly away from a stop sign. Is the work done on the car positive, negative, or zero?

Variable Force

We've learned how to calculate the work done on an object by a constant force, but what about a force that changes as the object moves? Equation 9.14, the definition of work, is all we need:

$$W = \int_{s_i}^{s_f} F_s ds = \text{area under the force-versus-position graph} \quad (9.21)$$

(work done by a variable force)

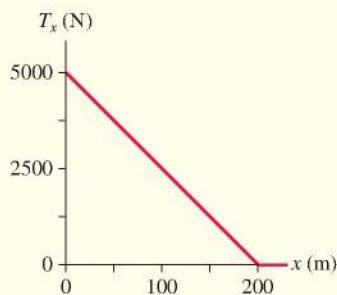
The integral sums up the small amounts of work $F_s ds$ done in each step along the trajectory. The only new feature, because F_s now varies with position, is that we cannot take F_s outside the integral. We must evaluate the integral either geometrically by finding the area under the curve (which we'll do in the next example) or by actually doing the integration (which we'll do in the next section).

EXAMPLE 9.7 Using work to find the speed of a car

A 1500 kg car is towed, starting from rest. **FIGURE 9.16** shows the tension force in the tow rope as the car travels from $x = 0$ m to $x = 200$ m. What is the car's speed after being pulled 200 m?

MODEL Let the system consist of only the car, which we model as a particle. We'll neglect rolling friction. Two vertical forces, the normal force and gravity, are perpendicular to the motion and thus do no work.

FIGURE 9.16 Force-versus-position graph for a car.



SOLVE We can solve this problem with the energy principle, $\Delta K = K_f - K_i = W$, where W is the work done by the tension force, but the force is not constant so we have to use the full definition of work as an integral. In this case, we can do the integral graphically:

$$\begin{aligned} W &= \int_{0 \text{ m}}^{200 \text{ m}} T_x dx \\ &= \text{area under the force curve from 0 m to 200 m} \\ &= \frac{1}{2}(5000 \text{ N})(200 \text{ m}) = 500,000 \text{ J} \end{aligned}$$

The initial kinetic energy is zero, so the final kinetic energy is simply the energy transferred to the system by the work of the tension: $K_f = W = 500,000 \text{ J}$. Then, from the definition of kinetic energy,

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(500,000 \text{ J})}{1500 \text{ kg}}} = 26 \text{ m/s}$$

ASSESS 26 m/s \approx 55 mph is a reasonable final speed after being towed 200 m.

9.4 Restoring Forces and the Work Done by a Spring

If you stretch a rubber band, a force tries to pull the rubber band back to its equilibrium, or unstretched, length. A force that restores a system to an equilibrium position is called a **restoring force**. Objects that exert restoring forces are called **elastic**. The most basic examples of elasticity are things like springs and rubber bands, but other examples of elasticity and restoring forces abound. For example, the steel beams flex slightly as you drive your car over a bridge, but they are restored to equilibrium after your car passes by. Nearly everything that stretches, compresses, flexes, bends, or twists exhibits a restoring force and can be called elastic.

We didn't introduce restoring forces in Part I of this textbook because we didn't have the mathematical tools to deal with them. But now—using work and energy—we do. We're going to use a simple spring as our model of elasticity. Suppose you have a spring whose **equilibrium length** is L_0 . This is the length of the spring when it is neither pushing nor pulling. If you stretch (or compress) the spring, how hard does it pull (or push) back? Measurements show that

- The force is *opposite the displacement*. This is what we *mean* by a restoring force.
- If you don't stretch or compress the spring too much, the force is *proportional to the displacement from equilibrium*. The farther you push or pull, the larger the force.

FIGURE 9.17 shows a spring along a generic s -axis exerting force \vec{F}_{Sp} . Notice that s_{eq} is the position, or coordinate, of the free end of the spring, *not* the spring's equilibrium length L_0 . When the spring is stretched, the **spring displacement** $\Delta s = s - s_{\text{eq}}$ is positive while $(F_{\text{Sp}})_s$, the s -component of the restoring force, is negative. Similarly, compressing the spring makes $\Delta s < 0$ and $(F_{\text{Sp}})_s > 0$. The graph of force versus displacement is a straight line with negative slope, showing that the spring force is proportional to but *opposite* the displacement.

The equation of the straight-line graph passing through the origin is

$$(F_{\text{Sp}})_s = -k \Delta s \quad (\text{Hooke's law}) \quad (9.22)$$

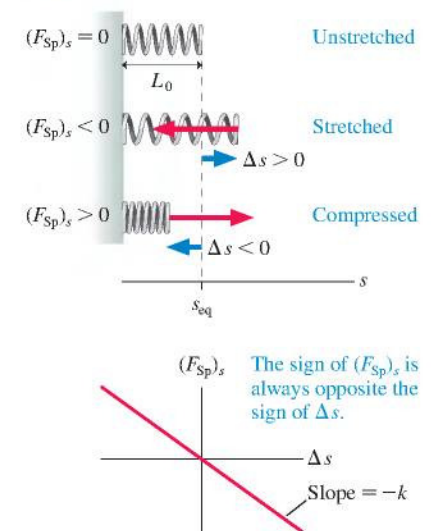
The minus sign is the mathematical indication of a *restoring* force, and the constant k —the absolute value of the slope of the line—is called the **spring constant** of the spring. The units of the spring constant are N/m. This relationship between the force and displacement of a spring was discovered by Robert Hooke, a contemporary (and sometimes bitter rival) of Newton. **Hooke's law** is not a true “law of nature,” in the sense that Newton's laws are, but is actually just a *model* of a restoring force. It works well for *small* displacements from equilibrium, but Hooke's law will fail for any real spring that is compressed or stretched too far. A hypothetical massless spring for which Hooke's law is true at all displacements is called an **ideal spring**.

NOTE The force does not depend on the spring's physical length L but, instead, on the *displacement* Δs of the end of the spring.

The spring constant k is a property that characterizes a spring, just as mass m characterizes a particle. For a given spring, k is a constant—it does not change as the spring is stretched or compressed. If k is large, it takes a large pull to cause a significant stretch, and we call the spring a “stiff” spring. A spring with small k can be stretched with very little force, and we call it a “soft” spring.

NOTE In an earlier physics course, you may have learned Hooke's law as $F_{\text{Sp}} = -kx$ rather than as $-k \Delta s$. This can be misleading, and it is a common source of errors. The restoring force is $-kx$ *only* if the coordinate system in the problem is chosen such that the origin is at the equilibrium position of the free end of the spring. That is, $x = \Delta s$ only if $x_{\text{eq}} = 0$. This choice of origin is often made, but in some problems it will be more convenient to locate the origin of the coordinate system elsewhere.

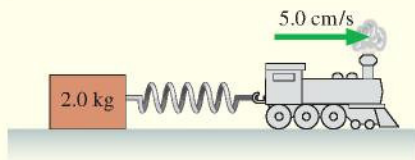
FIGURE 9.17 Properties of a spring.



EXAMPLE 9.8 Pull until it slips

FIGURE 9.18 shows a spring attached to a 2.0 kg block. The other end of the spring is pulled by a motorized toy train that moves forward at 5.0 cm/s. The spring constant is 50 N/m, and the coefficient of static friction between the block and the surface is 0.60. The spring is at its equilibrium length at $t = 0$ s when the train starts to move. When does the block slip?

FIGURE 9.18 A toy train stretches the spring until the block slips.

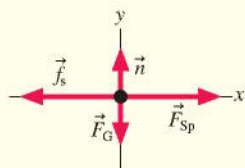


MODEL Model the block as a particle and the spring as an ideal spring obeying Hooke's law.

VISUALIZE **FIGURE 9.19** is a free-body diagram for the block.

SOLVE Recall that the tension in a massless string pulls equally at *both* ends of the string. The same is true

FIGURE 9.19 The free-body diagram.



for the spring force: It pulls (or pushes) equally at *both* ends. This is the key to solving the problem. As the right end of the spring moves, stretching the spring, the spring pulls backward on the train *and* forward on the block with equal strength. As the spring stretches, the static friction force on the block increases in magnitude to keep the block at rest. The block is in static equilibrium, so

$$\sum (F_{\text{net}})_x = (F_{\text{Sp}})_x + (f_s)_x = F_{\text{Sp}} - f_s = 0$$

where F_{Sp} is the *magnitude* of the spring force. The magnitude is $F_{\text{Sp}} = k \Delta x$, where $\Delta x = v_x t$ is the distance the train has moved. Thus

$$f_s = F_{\text{Sp}} = k \Delta x$$

The block slips when the static friction force reaches its maximum value $f_{s \text{ max}} = \mu_s n = \mu_s mg$. This occurs when the train has moved

$$\begin{aligned} \Delta x &= \frac{f_{s \text{ max}}}{k} = \frac{\mu_s mg}{k} = \frac{(0.60)(2.0 \text{ kg})(9.80 \text{ m/s}^2)}{50 \text{ N/m}} \\ &= 0.235 \text{ m} = 23.5 \text{ cm} \end{aligned}$$

The time at which the block slips is

$$t = \frac{\Delta x}{v_x} = \frac{23.5 \text{ cm}}{5.0 \text{ cm/s}} = 4.7 \text{ s}$$

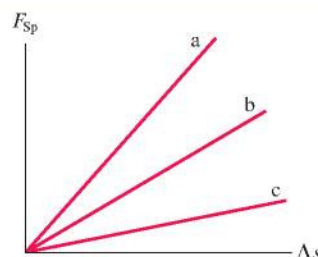


The slip can range from a few centimeters in a relatively small earthquake to several meters in a very large earthquake.

This example illustrates a class of motion called *stick-slip motion*. Once the block slips, it will shoot forward some distance, then stop and stick again. As the train continues, there will be a recurring sequence of stick, slip, stick, slip, stick. . . .

Earthquakes are an important example of stick-slip motion. The large tectonic plates making up the earth's crust are attempting to slide past each other, but friction causes the edges of the plates to stick together. You may think of rocks as rigid and brittle, but large masses of rock are somewhat elastic and can be "stretched." Eventually the elastic force of the deformed rocks exceeds the friction force between the plates. An earthquake occurs as the plates slip and lurch forward. Once the tension is released, the plates stick together again and the process starts all over.

STOP TO THINK 9.7 The graph shows the force magnitude versus displacement for three springs. Rank in order, from largest to smallest, the spring constants k_a , k_b , and k_c .

**Work Done by Springs**

The primary goal of this section is to calculate the work done by a spring. **FIGURE 9.20** shows a spring acting on an object as it moves from s_i to s_f . The spring force on the object varies as the object moves, but we can calculate the spring's work by using Equation 9.21 for a variable force. Hooke's law for the spring is $(F_{\text{Sp}})_s = -k \Delta s = -k(s - s_{\text{eq}})$. Thus

$$W = \int_{s_i}^{s_f} (F_{\text{Sp}})_s ds = -k \int_{s_i}^{s_f} (s - s_{\text{eq}}) ds \quad (9.23)$$

This is an integration best carried out with a change of variables. Define $u = s - s_{\text{eq}}$, in which case $ds = du$. This changes the integrand from $(s - s_{\text{eq}}) ds$ to $u du$. When we change variables, we also have to change the integration limits. At the lower limit, where $s = s_i$, the new variable u is $s_i - s_{\text{eq}} = \Delta s_i$. The lower limit becomes the initial displacement. Similarly, $s = s_f$ makes $u = s_f - s_{\text{eq}} = \Delta s_f$ at the upper limit. With these changes, the integral is

$$W = -k \int_{\Delta s_i}^{\Delta s_f} u ds = -\frac{1}{2}ku^2 \Big|_{\Delta s_i}^{\Delta s_f} = -\frac{1}{2}k(\Delta s_f)^2 + \frac{1}{2}k(\Delta s_i)^2 \quad (9.24)$$

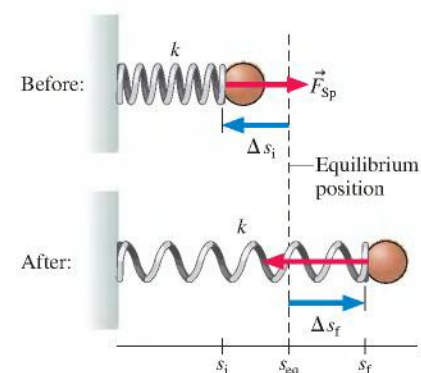
With a small rearrangement of the right side, we see that the work done by a spring is

$$W = -\left(\frac{1}{2}k(\Delta s_f)^2 - \frac{1}{2}k(\Delta s_i)^2\right) \quad (\text{work done by a spring}) \quad (9.25)$$

Because the displacements are squared, it makes no difference whether the initial and final displacements are stretches or compressions.

The work done by a spring is energy transferred to the object by the force of the spring. We can use this—and the energy principle—to solve problems that we were unable to solve with a direct application of Newton's laws.

FIGURE 9.20 The spring does work on the object.



EXAMPLE 9.9 Using the energy principle for a spring

The “pincube machine” was an ill-fated predecessor of the pinball machine. A 100 g cube is launched by pulling a spring back 12 cm and releasing it. The spring's spring constant is 65 N/m. What is the cube's launch speed as it leaves the spring? Assume that the surface is frictionless.

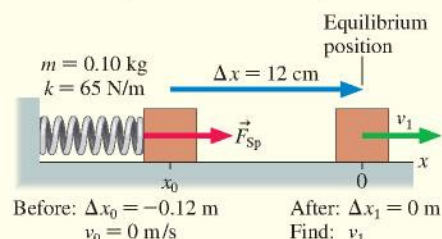
MODEL Let the system consist of only the cube, which we model as a particle. Two vertical forces, the normal force and gravity, are perpendicular to the cube's displacement, and we've seen that perpendicular forces do no work. Only the spring force does work.

VISUALIZE **FIGURE 9.21** is a before-and-after pictorial representation in which, for horizontal motion, we've replaced the generic s -axis with an x -axis. Notice that for problem solving we use numerical subscripts in place of the generic i and f .

SOLVE We can solve this problem with the energy principle, $\Delta K = K_1 - K_0 = W$, where W is the work done by the spring. The initial displacement is $\Delta x_0 = -0.12$ m. The cube will separate from the spring when the spring has expanded back to its equilibrium length, so the final displacement is $\Delta x_1 = 0$ m. From Equation 9.25, the spring does work

$$W = -\left(\frac{1}{2}k(\Delta x_1)^2 - \frac{1}{2}k(\Delta x_0)^2\right) = \frac{1}{2}(65 \text{ N/m})(-0.12 \text{ m})^2 - 0 = 0.468 \text{ J}$$

FIGURE 9.21 Pictorial representation of the pincube machine.

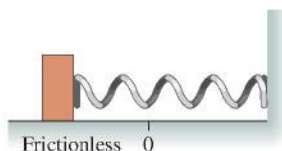


The initial kinetic energy is zero, so the final kinetic energy is simply the energy transferred to the system by the work of the spring: $K_1 = W = 0.468$ J. Then, from the definition of kinetic energy,

$$v_1 = \sqrt{\frac{2K_1}{m}} = \sqrt{\frac{2(0.468 \text{ J})}{0.10 \text{ kg}}} = 3.1 \text{ m/s}$$

ASSESS 3.1 m/s \approx 6 mph seems a reasonable final speed for a small, spring-launched cube.

STOP TO THINK 9.8 A block is attached to a spring, the spring is stretched, and the block is released at the position shown. As the block moves to the right, is the work done by the wall positive, negative, or zero?



9.5 Dissipative Forces and Thermal Energy

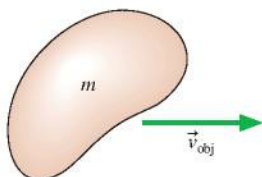
Suppose you drag a heavy sofa across the floor at a steady speed. You are doing work, but the sofa is not gaining kinetic energy. And when you stop pulling, the sofa almost instantly stops moving. Where is the energy going that you're adding to the system? And what happens to the sofa's kinetic energy when you stop pulling?

You know that rubbing things together raises their temperature, in extreme cases making them hot enough to start a fire. As the sofa slides across the floor, friction causes the bottom of the sofa and the floor to get hotter. An increasing temperature is associated with increasing *thermal energy*, so in this situation the work done by pulling is increasing the system's thermal energy instead of its kinetic energy. Our goal in this section is to understand what thermal energy is and how it is related to *dissipative forces*.

Energy at the Microscopic Level

FIGURE 9.22 Two perspectives of motion and energy.

(a) The macroscopic motion of the system as a whole



(b) The microscopic motion of the atoms inside

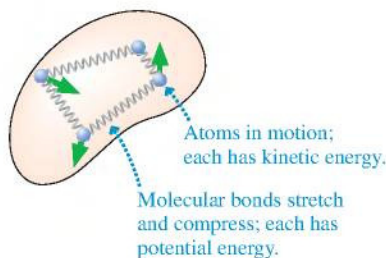


FIGURE 9.22 shows two different perspectives of an object. In the macrophysics perspective of Figure 9.22a you see an object of mass m moving as a whole with velocity v_{obj} . As a consequence of its motion, the object has macroscopic kinetic energy $K_{\text{macro}} = \frac{1}{2}mv_{\text{obj}}^2$.

NOTE You recognize the prefix *micro*, meaning “small.” You may not be familiar with *macro*, which means “large.” Everyday objects, which consist of vast numbers of particle-like atoms, are *macroscopic objects*. We will use the term **macrophysics** to refer to the motion and dynamics of an object as a whole and **microphysics** to refer to the motions of atoms within an object.

Figure 9.22b is a microphysics view of the same object, where now we see a *system of particles*. Each of these atoms is jiggling about and has kinetic energy. As the atoms move, they stretch and compress the spring-like bonds between them. We'll study potential energy in Chapter 10, but you'll recall from the energy overview at the beginning of this chapter that potential energy is *stored energy*. Stretched and compressed springs store energy, so the bonds have potential energy.

The kinetic energy of one atom is exceedingly small, but there are enormous numbers of atoms in a macroscopic object. The total kinetic energy of all the atoms is what we call the *microscopic kinetic energy*, K_{micro} . The total potential energy of all the bonds is the *microscopic potential energy*, U_{micro} . These energies are distinct from the macroscopic energy of the object as a whole.

The combined microscopic kinetic and potential energy of the atoms—the energy of the jiggling atoms and stretching bonds—is called the **thermal energy** of the system:

$$E_{\text{th}} = K_{\text{micro}} + U_{\text{micro}} \quad (9.26)$$

This energy is hidden from view in our macrophysics perspective, but it is quite real. We will discover later, when we reach thermodynamics, that the thermal energy is related to the *temperature* of the system. Raising the temperature causes the atoms to move faster and the bonds to stretch more, giving the system more thermal energy.

With the inclusion of thermal energy, the system has both macroscopic kinetic energy *and* thermal energy: $E_{\text{sys}} = K + E_{\text{th}}$. K is understood to be the total macroscopic kinetic energy; we'll use the subscript “macro” only if there's a need to distinguish macroscopic energy from microscopic energy. With this, the energy principle becomes

$$\Delta E_{\text{sys}} = \Delta K + \Delta E_{\text{th}} = W_{\text{ext}} \quad (9.27)$$

Work done on the system might increase the system's kinetic energy, its thermal energy, or both. Or, in the absence of work, kinetic energy can be transformed into thermal energy as long as the total energy change is zero. Recognizing thermal energy greatly expands the range of problems we can analyze with the energy principle.

NOTE The microscopic energy of atoms is *not* called “heat.” As we've already seen, heat is a *process*, similar to work, for transferring energy between the system and the environment. We'll have a lot more to say about heat in future chapters. For the time being we want to use the correct term “thermal energy” to describe the random, thermal motions of the atoms in a system. If the temperature of a system goes up (i.e., it gets hotter), it is because the system's thermal energy has increased.

Dissipative Forces

Forces such as friction and drag cause the macroscopic kinetic energy of a system to be *dissipated* as thermal energy. Hence these are called **dissipative forces**. FIGURE 9.23 shows how microscopic interactions are responsible for transforming macroscopic kinetic energy into thermal energy when two objects slide against each other. Because friction causes *both* objects to get warmer, with increased thermal energy, we must define the system to include both objects whose temperature changes—both the sofa and the floor.

For example, FIGURE 9.24 shows a box being pulled at constant speed across a horizontal surface with friction. As you can imagine, both the surface and the box are getting warmer—increasing thermal energy—but the kinetic energy is not changing. If we define the system to be box + surface, then the increasing thermal energy of the system is entirely due to the work being done on the system by tension in the rope:

$$\Delta E_{\text{th}} = W_{\text{tension}}$$

The work done by tension in pulling the box through distance Δs is simply $W_{\text{tension}} = T\Delta s$; thus $\Delta E_{\text{th}} = T\Delta s$. Because the box is moving with constant velocity, and thus no net force, the tension force has to exactly balance the friction force: $T = f_k$. Consequently, the increase in thermal energy due to the dissipative force of friction is

$$\Delta E_{\text{th}} = f_k \Delta s \quad (9.28)$$

Notice that the increase in thermal energy is directly proportional to the total distance of sliding. **Dissipative forces always increase the thermal energy; they never decrease it.**

You might wonder why we didn't simply calculate the work done by friction. The rather subtle reason is that work is defined only for forces acting on a *particle*. There is work being done on individual atoms at the boundary as they are pulled this way and that, but we would need a detailed knowledge of atomic-level friction forces to calculate this work. The friction force \vec{f}_k is an average force on the object as a whole; it is not a force on any particular particle, so we cannot use it to calculate work. Furthermore, increasing thermal energy is not an energy transfer—the definition of work—from the box to the surface or from the surface to the box; both the box and the surface are gaining thermal energy. **The techniques used to calculate the work done on a particle cannot be used to calculate the work done by dissipative forces.**

NOTE The considerations that led to Equation 9.28 allow us to calculate the total increase in thermal energy of the entire system, but we cannot determine what fraction of ΔE_{th} goes to the box and what fraction goes to the surface.

FIGURE 9.23 Motion with friction leads to thermal energy.

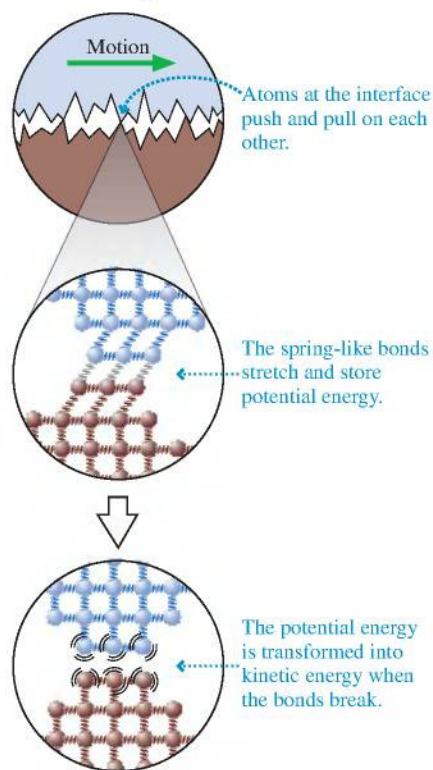
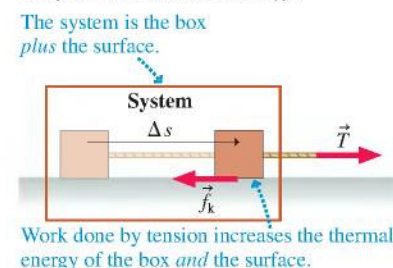


FIGURE 9.24 Work done by tension is dissipated as thermal energy.



EXAMPLE 9.10 Increasing kinetic and thermal energy

A rope with 30 N of tension pulls a 10 kg crate 3.0 m across a horizontal floor, starting from rest. The coefficient of friction between the crate and the floor is 0.20. What is the increase in thermal energy? What is the crate's final speed?

MODEL Let the system consist of both the crate and the floor. The tension in the rope does work on the system, but the vertical normal force and gravitational force do not.

SOLVE The energy principle, Equation 9.27, is $\Delta K + \Delta E_{\text{th}} = W_{\text{ext}}$. The friction force on an object moving on a horizontal surface is $f_k = \mu_k n = \mu_k mg$, so the increase in thermal energy, given by Equation 9.28, is

$$\begin{aligned} \Delta E_{\text{th}} &= f_k \Delta s = \mu_k mg \Delta s \\ &= (0.20)(10 \text{ kg})(9.80 \text{ m/s}^2)(3.0 \text{ m}) = 59 \text{ J} \end{aligned}$$

The tension force does work $W_{\text{ext}} = T\Delta s = (30 \text{ N})(3.0 \text{ m}) = 90 \text{ J}$. 59 J of this goes to increasing the thermal energy, so $\Delta K = 31 \text{ J}$ is the crate's change of kinetic energy. Because $K_i = 0 \text{ J}$, $K_f = \Delta K = 31 \text{ J}$. Using the definition of kinetic energy, we find that the crate's final speed is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(31 \text{ J})}{10 \text{ kg}}} = 2.5 \text{ m/s}$$

ASSESS The thermal energy of the crate and floor increases by 59 J. We cannot determine ΔE_{th} for the crate (or floor) alone.

9.6 Power

Work is a transfer of energy between the environment and a system. In many situations we would like to know *how quickly* the energy is transferred. Does the force act quickly and transfer the energy very rapidly, or is it a slow and lazy transfer of energy? If you need to buy a motor to lift 1000 kg of bricks up 20 m, it makes a *big* difference whether the motor has to do this in 30 s or 30 min!

The question *How quickly?* implies that we are talking about a *rate*. For example, the velocity of an object—how quickly it is moving—is the *rate of change* of position. So when we raise the issue of how quickly the energy is transferred, we are talking about the *rate of transfer* of energy. The rate at which energy is transferred or transformed is called the **power** P , and it is defined as

$$P = \frac{dE_{\text{sys}}}{dt} \quad (9.29)$$

The unit of power is the **watt**, which is defined as 1 watt = 1 W = 1 J/s. Common prefixes used with power are mW (milliwatts), kW (kilowatts), and MW (megawatts).

For example, the rope in Example 9.10 pulled with a tension of 30 N and, by doing work, transferred 90 J of energy to the system. If it took 10 s to drag the crate 3.0 m, then energy was being transferred at the rate of 9 J/s. We would say that whatever was supplying this energy—whether a human or a motor—has a “power output” of 9 W.

The idea of power as a *rate* of energy transfer applies no matter what the form of energy. FIGURE 9.25 shows three examples of the idea of power. For now, we want to focus primarily on *work* as the source of energy transfer. Within this more limited scope, power is simply the **rate of doing work**: $P = dW/dt$. If a particle moves through a small displacement $d\vec{r}$ while acted on by force \vec{F} , the force does a small amount of work dW given by

$$dW = \vec{F} \cdot d\vec{r}$$

Dividing both sides by dt , to give a rate of change, yields

$$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

But $d\vec{r}/dt$ is the velocity \vec{v} , so we can write the power as

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta \quad (9.30)$$

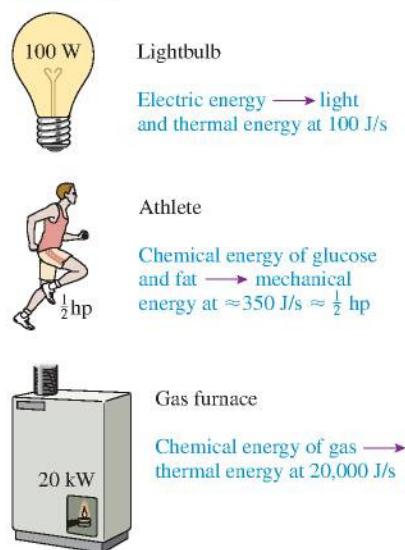
In other words, the power delivered to a particle by a force acting on it is the dot product of the force and the particle’s velocity. These ideas will become clearer with some examples.

The English unit of power is the *horsepower*. The conversion factor to watts is

$$1 \text{ horsepower} = 1 \text{ hp} = 746 \text{ W}$$

Many common appliances, such as motors, are rated in hp.

FIGURE 9.25 Examples of power.



EXAMPLE 9.11 Power output of a motor

A factory uses a motor and a cable to drag a 300 kg machine to the proper place on the factory floor. What power must the motor supply to drag the machine at a speed of 0.50 m/s? The coefficient of friction between the machine and the floor is 0.60.

SOLVE The force applied by the motor, through the cable, is the tension force \vec{T} . This force does work on the machine with power $P = Tv$. The machine is in equilibrium, because the motion is at

constant velocity, hence the tension in the rope balances the friction and is

$$T = f_k = \mu_k mg$$

The motor’s power output is

$$P = Tv = \mu_k mgv = 882 \text{ W}$$

EXAMPLE 9.12 Power output of a car engine

A 1500 kg car has a front profile that is 1.6 m wide by 1.4 m high and a drag coefficient of 0.50. The coefficient of rolling friction is 0.02. What power must the engine provide to drive at a steady 30 m/s (≈ 65 mph) if 25% of the power is “lost” before reaching the drive wheels?

SOLVE The net force on a car moving at a steady speed is zero. The motion is opposed both by rolling friction and by air resistance. The forward force on the car \vec{F}_{car} (recall that this is really $\vec{F}_{\text{ground on car}}$, a

reaction to the drive wheels pushing backward on the ground with $\vec{F}_{\text{car on ground}}$) exactly balances the two opposing forces:

$$F_{\text{car}} = f_r + F_{\text{drag}}$$

where \vec{F}_{drag} is the drag due to the air. Using the results of Chapter 6, where both rolling friction and drag were introduced, this becomes

$$F_{\text{car}} = \mu_r mg + \frac{1}{2} C_p A v^2 = 294 \text{ N} + 655 \text{ N} = 949 \text{ N}$$

Here $A = (1.6 \text{ m}) \times (1.4 \text{ m})$ is the front cross-section area of the car, and we used 1.3 kg/m^3 as the density of air. The power required to push the car forward at 30 m/s is

$$P_{\text{car}} = F_{\text{car}} v = (949 \text{ N})(30 \text{ m/s}) = 28,500 \text{ W} = 38 \text{ hp}$$

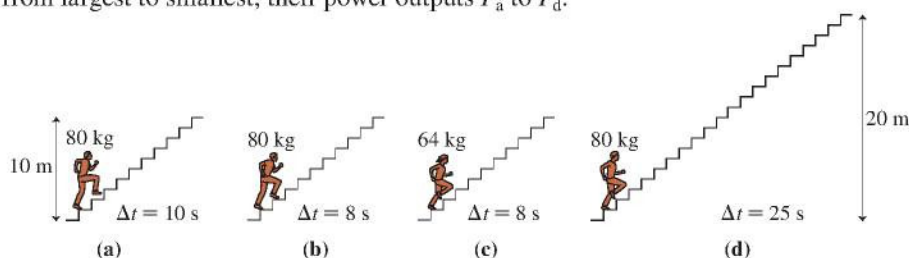
This is the power *needed* at the drive wheels to push the car against the dissipative forces of friction and air resistance. The power output

of the engine is larger because some energy is used to run the water pump, the power steering, and other accessories. In addition, energy is lost to friction in the drive train. If 25% of the power is lost (a typical value), leading to $P_{\text{car}} = 0.75 P_{\text{engine}}$, the engine's power output is

$$P_{\text{engine}} = \frac{P_{\text{car}}}{0.75} = 38,000 \text{ W} = 51 \text{ hp}$$

ASSESS Automobile engines are typically rated at $\approx 200 \text{ hp}$. Most of that power is reserved for fast acceleration and climbing hills.

STOP TO THINK 9.9 Four students run up the stairs in the time shown. Rank in order, from largest to smallest, their power outputs P_a to P_d .



CHALLENGE EXAMPLE 9.13 Stopping a brick

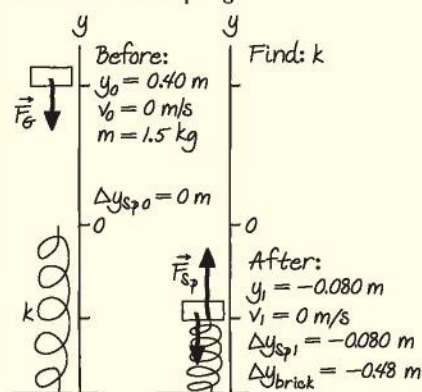
A 25.0-cm-long spring stands vertically on the ground, with its lower end secured in a base. A 1.5 kg brick is held 40 cm directly above the spring and dropped onto the spring. The spring compresses to a length of 17.0 cm before starting to launch the brick back upward. What is the spring's spring constant?

MODEL Let the system consist of only the brick, which is modeled as a particle. Assume that the spring is ideal. Both gravity and, after contact, the spring do work on the brick.

VISUALIZE FIGURE 9.26 is a before-and-after pictorial representation. We've chosen to place the origin of the y -axis at the equilibrium position of the spring's upper end. The length of the spring is not relevant, but the *difference* between its before and after lengths—8.0 cm—is the spring's maximum compression. The point of maximum compression is a turning point for the brick, as it reverses direction and starts moving back up, so the brick's instantaneous velocity is zero.

SOLVE At first, this seems like a two-part problem: free fall until hitting the spring, then deceleration as the spring compresses. But by using the energy principle, $\Delta E_{\text{sys}} = \Delta K = W_{\text{tot}} = W_G + W_{\text{Sp}}$,

FIGURE 9.26 Pictorial representation of the brick and spring.



we can do it in one step. Interestingly, $\Delta K = 0$ because the brick is instantaneously at rest at the beginning and again at the point of maximum spring compression. Consequently, $W_{\text{tot}} = 0$. That's not a difficulty because gravity does positive work (the downward gravitational force is in the direction of the brick's displacement) while

the spring does negative work (the upward spring force is opposite the displacement).

The work done by gravity is

$$W_G = (F_G)_y \Delta y_{\text{brick}} = -mg \Delta y_{\text{brick}}$$

where Δy_{brick} is the brick's total displacement. The negative sign comes from $(F_G)_y = -mg$, but W_G is positive because $\Delta y_{\text{brick}} = y_1 - y_0 = -0.48 \text{ m}$ is also negative. It may seem strange that calculating the work done by gravity is so simple when the brick first accelerates, then slows quickly after hitting the spring. But work depends on only the displacement, not how fast or slow the object is moving.

We have to be careful with the spring because its displacement is not the same as the brick's displacement. The spring begins compressing only when contact is made, so $\Delta y_{\text{Sp}0} = 0 \text{ m}$ at the instant the brick is released. The work done by the spring then continues until maximum compression, when the spring's displacement is $\Delta y_{\text{Sp}1} = -0.080 \text{ m}$. The work done by the spring, Equation 9.25, is thus

$$W_{\text{Sp}} = -\left(\frac{1}{2}k(\Delta y_{\text{Sp}1})^2 - \frac{1}{2}k(\Delta y_{\text{Sp}0})^2\right) = -\frac{1}{2}k(\Delta y_{\text{Sp}1})^2$$

With this information about the two works, the energy principle is

$$\Delta K = 0 = W_G + W_{\text{Sp}} = -mg \Delta y_{\text{brick}} - \frac{1}{2}k(\Delta y_{\text{Sp}1})^2$$

Solving for the spring constant gives

$$k = \frac{2mg \Delta y_{\text{brick}}}{(\Delta y_{\text{Sp}1})^2} = \frac{2(1.5 \text{ kg})(9.80 \text{ m/s}^2)(-0.48 \text{ m})}{(-0.080 \text{ m})^2} = 2200 \text{ N/m}$$

ASSESS 2200 N/m is a fairly large spring constant, but that's to be expected for a spring that's going to stop a falling ≈ 3 pound brick. The complexity of this problem was not the math, which was fairly simple, but the reasoning. It's a good illustration of how to apply energy reasoning to other problems.

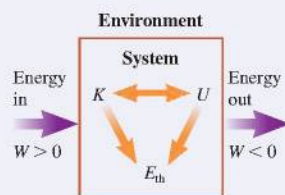
SUMMARY

The goal of Chapter 9 has been to begin your study of how energy is transferred and transformed.

GENERAL PRINCIPLES

Basic Energy Model

- Energy is a property of the system.
- Energy is *transformed* within the system without loss.
- Energy is *transferred* to and from the system by forces that do work W .
- $W > 0$ for energy added.
- $W < 0$ for energy removed.



The Energy Principle

Doing work on a system changes the system energy:

$$\Delta E_{\text{sys}} = W_{\text{ext}}$$

For systems containing only particles, no interactions, $E_{\text{sys}} = K + E_{\text{th}}$. All forces are external forces, so

$$\Delta K + \Delta E_{\text{th}} = W_{\text{tot}}$$

where W_{tot} is the total work done on all particles.

IMPORTANT CONCEPTS

Kinetic energy is an energy of motion: $K = \frac{1}{2}mv^2$

Potential energy is stored energy.

Thermal energy is the **microscopic** energy of moving atoms and stretched bonds.

Dissipative forces, such as friction and drag, transform **macroscopic** energy into thermal energy. For friction:

$$\Delta E_{\text{th}} = f_k \Delta s$$

The **work** done by a force on a particle as it moves from s_i to s_f is

$$W = \int_{s_i}^{s_f} F_s ds = \text{area under the force curve}$$

The work done by a constant force is

$$W = \vec{F} \cdot \Delta \vec{r}$$

The work done by a spring is

$$W = -\left(\frac{1}{2}k(\Delta s_f)^2 - \frac{1}{2}k(\Delta s_i)^2\right)$$

where Δs is the displacement of the end of the spring.

APPLICATIONS

Hooke's law

The restoring force of an ideal spring is

$$(F_{\text{sp}})_s = -k \Delta s$$

where k is the **spring constant** and Δs is the displacement of the end of the spring from equilibrium.



Power is the rate at which energy is transferred or transformed:

$$P = dE_{\text{sys}}/dt$$

For a particle with velocity \vec{v} , the power delivered to the particle by force \vec{F} is $P = \vec{F} \cdot \vec{v} = Fv \cos \theta$.

Dot product

$$\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y$$



TERMS AND NOTATION

energy	energy transformation	scalar product	ideal spring
system	energy principle	restoring force	macrophysics
environment	basic energy model	elastic	microphysics
system energy, E_{sys}	before-and-after representation	equilibrium length, L_0	thermal energy, E_{th}
work, W	kinetic energy, K	spring displacement, Δs	dissipative force
heat	joule, J	spring constant, k	power, P
energy transfer	dot product	Hooke's law	watt, W

CONCEPTUAL QUESTIONS

- If a particle's speed increases by a factor of 3, by what factor does its kinetic energy change?
- Particle A has half the mass and eight times the kinetic energy of particle B. What is the speed ratio v_A/v_B ?
- An elevator held by a single cable is ascending but slowing down. Is the work done by tension positive, negative, or zero? What about the work done by gravity? Explain.
- The rope in **FIGURE Q9.4** pulls the box to the left across a rough surface. Is the work done by tension positive, negative, or zero? Explain.
- A 0.2 kg plastic cart and a 20 kg lead cart both roll without friction on a horizontal surface. Equal forces are used to push both carts forward a distance of 1 m, starting from rest. After traveling 1 m, is the kinetic energy of the plastic cart greater than, less than, or equal to the kinetic energy of the lead cart? Explain.
- A particle moving to the left is slowed by a force pushing to the right. Is the work done on the particle positive or negative? Or is there not enough information to tell? Explain.
- A particle moves in a vertical plane along the *closed* path seen in **FIGURE Q9.7**, starting at A and eventually returning to its starting point. Is the work done by gravity positive, negative, or zero? Explain.



FIGURE Q9.4

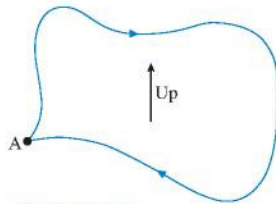


FIGURE Q9.7

- You need to raise a heavy block by pulling it with a massless rope. You can either (a) pull the block straight up height h , or (b) pull it up a long, frictionless plane inclined at a 15° angle until its height has increased by h . Assume you will move the block at constant speed either way. Will you do more work in case a or case b? Or is the work the same in both cases? Explain.
- A ball on a string travels once around a circle with a circumference of 2.0 m. The tension in the string is 5.0 N. How much work is done by tension?
- A sprinter accelerates from rest. Is the work done on the sprinter positive, negative, or zero? Explain.
- A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.
 - For what length of the spring is its restoring force $3F$?
 - At what compressed length is the restoring force $2F$?
- The left end of a spring is attached to a wall. When Bob pulls on the right end with a 200 N force, he stretches the spring by 20 cm. The same spring is then used for a tug-of-war between Bob and Carlos. Each pulls on his end of the spring with a 200 N force. How far does the spring stretch? Explain.
- The driver of a car traveling at 60 mph slams on the brakes, and the car skids to a halt. What happened to the kinetic energy the car had just before stopping?
- The motor of a crane uses power P to lift a steel beam. By what factor must the motor's power increase to lift the beam twice as high in half the time?

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 9.2 Work and Kinetic Energy for a Single Particle

- Which has the larger kinetic energy, a 10 g bullet fired at 500 m/s or a 75 kg student running at 5.5 m/s?
- At what speed does a 1000 kg compact car have the same kinetic energy as a 20,000 kg truck going 25 km/h?
- A mother has four times the mass of her young son. Both are running with the same kinetic energy. What is the ratio $v_{\text{son}}/v_{\text{mother}}$ of their speeds?
- A horizontal rope with 15 N tension drags a 25 kg box 2.0 m to the left across a horizontal surface. How much work is done by (a) tension and (b) gravity?
- A 25 kg box sliding to the left across a horizontal surface is brought to a halt in a distance of 35 cm by a horizontal rope pulling to the right with 15 N tension. How much work is done by (a) tension and (b) gravity?
- A 2.0 kg book is lying on a 0.75-m-high table. You pick it up and place it on a bookshelf 2.25 m above the floor.
 - How much work does gravity do on the book?
 - How much work does your hand do on the book?
- A 20 g particle is moving to the left at 30 m/s. A force on the particle causes it to move to the right at 30 m/s. How much work is done by the force?

- FIGURE EX9.8** is the kinetic-energy graph for a 2.0 kg object moving along the x -axis. Determine the work done on the object during each of the four intervals AB, BC, CD, and DE.

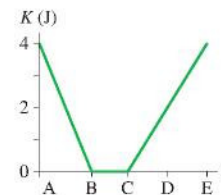


FIGURE EX9.8

- You throw a 5.5 g coin straight down at 4.0 m/s from a 35-m-high bridge.
 - How much work does gravity do as the coin falls to the water below?
 - What is the speed of the coin just as it hits the water?
- The cable of a crane is lifting a 750 kg girder. The girder increases its speed from 0.25 m/s to 0.75 m/s in a distance of 3.5 m.
 - How much work is done by gravity?
 - How much work is done by tension?

Section 9.3 Calculating the Work Done

- Evaluate the dot product $\vec{A} \cdot \vec{B}$ if
 - $\vec{A} = 4\hat{i} - 2\hat{j}$ and $\vec{B} = -2\hat{i} - 3\hat{j}$.
 - $\vec{A} = -4\hat{i} + 2\hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$.

12. I Evaluate the dot product $\vec{A} \cdot \vec{B}$ if
- $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 2\hat{i} - 6\hat{j}$.
 - $\vec{A} = 3\hat{i} - 2\hat{j}$ and $\vec{B} = 6\hat{i} + 4\hat{j}$.
13. I What is the angle θ between vectors \vec{A} and \vec{B} in each part of Exercise 12?
14. I Evaluate the dot product of the three pairs of vectors in FIGURE EX9.14.

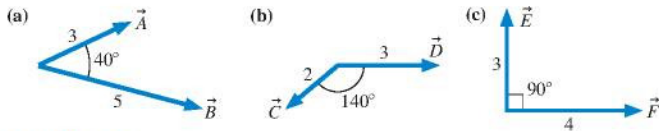


FIGURE EX9.14

15. I Evaluate the dot product of the three pairs of vectors in FIGURE EX9.15.

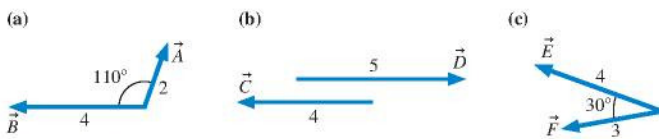


FIGURE EX9.15

16. II A 25 kg air compressor is dragged up a rough incline from $\vec{r}_1 = (1.3\hat{i} + 1.3\hat{j})$ m to $\vec{r}_2 = (8.3\hat{i} + 2.9\hat{j})$ m, where the y -axis is vertical. How much work does gravity do on the compressor during this displacement?
17. II A 45 g bug is hovering in the air. A gust of wind exerts a force $\vec{F} = (4.0\hat{i} - 6.0\hat{j}) \times 10^{-2}$ N on the bug.
- How much work is done by the wind as the bug undergoes displacement $\Delta\vec{r} = (2.0\hat{i} - 2.0\hat{j})$ m?
 - What is the bug's speed at the end of this displacement? Assume that the speed is due entirely to the wind.
18. II The two ropes seen in FIGURE EX9.18 are used to lower a 255 kg piano 5.00 m from a second-story window to the ground. How much work is done by each of the three forces?

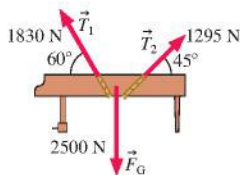


FIGURE EX9.18

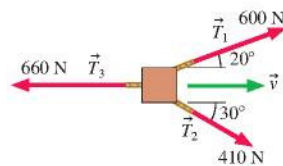


FIGURE EX9.19

19. II The three ropes shown in the bird's-eye view of FIGURE EX9.19 are used to drag a crate 3.0 m across the floor. How much work is done by each of the three forces?
20. I FIGURE EX9.20 is the force-versus-position graph for a particle moving along the x -axis. Determine the work done on the particle during each of the three intervals 0–1 m, 1–2 m, and 2–3 m.

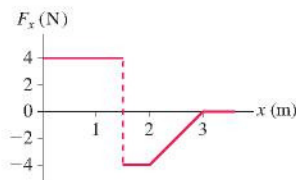


FIGURE EX9.20

21. II A 500 g particle moving along the x -axis experiences the force shown in FIGURE EX9.21. The particle's velocity is 2.0 m/s at $x = 0$ m. What is its velocity at $x = 3$ m?

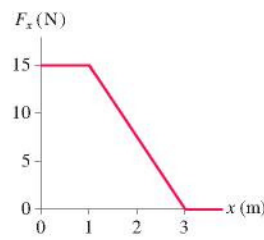


FIGURE EX9.21

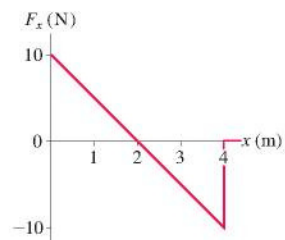


FIGURE EX9.22

22. II A 2.0 kg particle moving along the x -axis experiences the force shown in FIGURE EX9.22. The particle's velocity is 4.0 m/s at $x = 0$ m. What is its velocity at $x = 2$ m and $x = 4$ m?
23. II A particle moving on the x -axis experiences a force given by **CALC** $F_x = qx^2$, where q is a constant. How much work is done on the particle as it moves from $x = 0$ to $x = d$?
24. II A 150 g particle at $x = 0$ is moving at 2.00 m/s in the **CALC** $+x$ -direction. As it moves, it experiences a force given by $F_x = (0.250 \text{ N}) \sin(x/2.00 \text{ m})$. What is the particle's speed when it reaches $x = 3.14$ m?

Section 9.4 Restoring Forces and the Work Done by a Spring

25. I A horizontal spring with spring constant 750 N/m is attached to a wall. An athlete presses against the free end of the spring, compressing it 5.0 cm. How hard is the athlete pushing?
26. I A 35-cm-long vertical spring has one end fixed on the floor. Placing a 2.2 kg physics textbook on the spring compresses it to a length of 29 cm. What is the spring constant?
27. II A 10-cm-long spring is attached to the ceiling. When a 2.0 kg mass is hung from it, the spring stretches to a length of 15 cm.
- What is the spring constant?
 - How long is the spring when a 3.0 kg mass is suspended from it?
28. II A 60 kg student is standing atop a spring in an elevator as it accelerates upward at 3.0 m/s^2 . The spring constant is 2500 N/m. By how much is the spring compressed?
29. II A 5.0 kg mass hanging from a spring scale is slowly lowered onto a vertical spring, as shown in FIGURE EX9.29. The scale reads in newtons.
- What does the spring scale read just before the mass touches the lower spring?
 - The scale reads 20 N when the lower spring has been compressed by 2.0 cm. What is the value of the spring constant for the lower spring?
 - At what compression will the scale read zero?
30. II A horizontal spring with spring constant 85 N/m extends outward from a wall just above floor level. A 1.5 kg box sliding across a frictionless floor hits the end of the spring and compresses it 6.5 cm before the spring expands and shoots the box back out. How fast was the box going when it hit the spring?

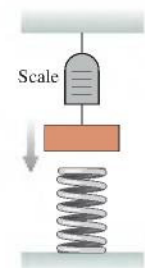


FIGURE EX9.29

Section 9.5 Dissipative Forces and Thermal Energy

31. I One mole (6.02×10^{23} atoms) of helium atoms in the gas phase has 3700 J of microscopic kinetic energy at room temperature. If we assume that all atoms move with the same speed, what is that speed? The mass of a helium atom is 6.68×10^{-27} kg.

32. I A 55 kg softball player slides into second base, generating 950 J of thermal energy in her legs and the ground. How fast was she running?
33. I A baggage handler throws a 15 kg suitcase along the floor of an airplane luggage compartment with a speed of 1.2 m/s. The suitcase slides 2.0 m before stopping. Use work and energy to find the suitcase's coefficient of kinetic friction on the floor.
34. III An 8.0 kg crate is pulled 5.0 m up a 30° incline by a rope angled 18° above the incline. The tension in the rope is 120 N, and the crate's coefficient of kinetic friction on the incline is 0.25.
- How much work is done by tension, by gravity, and by the normal force?
 - What is the increase in thermal energy of the crate and incline?
35. II Justin, with a mass of 30 kg, is going down an 8.0-m-high water slide. He starts at rest, and his speed at the bottom is 11 m/s. How much thermal energy is created by friction during his descent?

Section 9.6 Power

36. I
- How much work does an elevator motor do to lift a 1000 kg elevator a height of 100 m?
 - How much power must the motor supply to do this in 50 s at constant speed?
37. I
- How much work must you do to push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s?
 - What is your power output while doing so?
38. II How much energy is consumed by (a) a 1.2 kW hair dryer used for 10 min and (b) a 10 W night light left on for 24 h?
39. I At midday, solar energy strikes the earth with an intensity of about 1 kW/m^2 . What is the area of a solar collector that could collect 150 MJ of energy in 1 h? This is roughly the energy content of 1 gallon of gasoline.
40. III A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at constant acceleration.
- What is the magnitude of the horizontal force acting on the sprinter?
 - What is the sprinter's power output at 2.0 s, 4.0 s, and 6.0 s?
41. II A 70 kg human sprinter can accelerate from rest to 10 m/s in 3.0 s. During the same time interval, a 30 kg greyhound can go from rest to 20 m/s. What is the average power output of each? Average power over a time interval Δt is $\Delta E/\Delta t$.
42. II BIO The human heart pumps the average adult's 6.0 L (6000 cm^3) of blood through the body every minute. The heart must do work to overcome frictional forces that resist blood flow. The average adult blood pressure is $1.3 \times 10^4 \text{ N/m}^2$.
- How much work does the heart do to move the 6.0 L of blood completely through the body?
 - What power output must the heart have to do this task once a minute?
- Hint:** When the heart contracts, it applies force to the blood. Pressure is force/area. Model the circulatory system as a single closed tube, with cross-section area A and volume $V = 6.0 \text{ L}$, filled with blood to which the heart applies a force.
44. II
- Starting from rest, a crate of mass m is pushed up a frictionless slope of angle θ by a horizontal force of magnitude F . Use work and energy to find an expression for the crate's speed v when it is at height h above the bottom of the slope.
 - Doug uses a 25 N horizontal force to push a 5.0 kg crate up a 2.0-m-high, 20° frictionless slope. What is the speed of the crate at the top of the slope?
45. III Susan's 10 kg baby brother Paul sits on a mat. Susan pulls the mat across the floor using a rope that is angled 30° above the floor. The tension is a constant 30 N and the coefficient of friction is 0.20. Use work and energy to find Paul's speed after being pulled 3.0 m.
46. II A particle of mass m moving along the x -axis has velocity $v_x = v_0 \sin(\pi x/2L)$. How much work is done on the particle as it moves (a) from $x = 0$ to $x = L$ and (b) from $x = 0$ to $x = 2L$?
47. I A ball shot straight up with kinetic energy K_0 reaches height h . What height will it reach if the initial kinetic energy is doubled?
48. II A pile driver lifts a 250 kg weight and then lets it fall onto the end of a steel pipe that needs to be driven into the ground. A fall of 1.5 m drives the pipe in 35 cm. What is the average force exerted on the pipe?
49. II A 50 kg ice skater is gliding along the ice, heading due north at 4.0 m/s. The ice has a small coefficient of static friction, to prevent the skater from slipping sideways, but $\mu_k = 0$. Suddenly, a wind from the northeast exerts a force of 4.0 N on the skater.
- Use work and energy to find the skater's speed after gliding 100 m in this wind.
 - What is the minimum value of μ_s that allows her to continue moving straight north?
50. I You're fishing from a tall pier and have just caught a 1.5 kg fish. As it breaks the surface, essentially at rest, the tension in the vertical fishing line is 16 N. Use work and energy to find the fish's upward speed after you've lifted it 2.0 m.
51. II Hooke's law describes an ideal spring. Many real springs are better described by the restoring force $(F_{sp})_s = -k\Delta s - q(\Delta s)^3$, where q is a constant. Consider a spring with $k = 250 \text{ N/m}$ and $q = 800 \text{ N/m}^3$.
- How much work must you do to compress this spring 15 cm? Note that, by Newton's third law, the work you do on the spring is the negative of the work done by the spring.
 - By what percent has the cubic term increased the work over what would be needed to compress an ideal spring?
- Hint:** Let the spring lie along the s -axis with the equilibrium position of the end of the spring at $s = 0$. Then $\Delta s = s$.
52. II CALC The force acting on a particle is $F_x = F_0 e^{-\alpha/L}$. How much work does this force do as the particle moves along the x -axis from $x = 0$ to $x = L$?
53. II CALC The gravitational attraction between two objects with masses m_1 and m_2 , separated by distance x , is $F = Gm_1m_2/x^2$, where G is the gravitational constant.
- How much work is done by gravity when the separation changes from x_1 to x_2 ? Assume $x_2 < x_1$.
 - If one mass is much greater than the other, the larger mass stays essentially at rest while the smaller mass moves toward it. Suppose a $1.5 \times 10^{13} \text{ kg}$ comet is passing the orbit of Mars, heading straight for the sun at a speed of $3.5 \times 10^4 \text{ m/s}$. What will its speed be when it crosses the orbit of Mercury? Astronomical data are given in the tables at the back of the book, and $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

Problems

43. II A 1000 kg elevator accelerates upward at 1.0 m/s^2 for 10 m, starting from rest.
- How much work does gravity do on the elevator?
 - How much work does the tension in the elevator cable do on the elevator?
 - What is the elevator's kinetic energy after traveling 10 m?

54. || An *electric dipole* consists of two equal but opposite electric charges, $+q$ and $-q$, separated by a small distance d . If another charge Q is distance x from the center of the dipole, in the plane perpendicular to the line between $+q$ and $-q$, and if $x \gg d$, the dipole exerts an electric force $F = KqQd/x^3$ on charge Q , where K is a constant. How much work does the electric force do if charge Q moves from distance x_1 to distance x_2 ?

55. || A 50 g rock is placed in a slingshot and the rubber band is stretched. The magnitude of the force of the rubber band on the rock is shown by the graph in FIGURE P9.55. The rubber band is stretched 30 cm and then released. What is the speed of the rock?

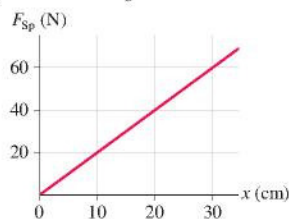


FIGURE P9.55

56. || When a 65 kg cheerleader stands on a vertical spring, the spring compresses by 5.5 cm. When a second cheerleader stands on the shoulders of the first, the spring compresses an additional 4.5 cm. What is the mass of the second cheerleader?

57. || Two identical horizontal springs are attached to opposite sides of a box that sits on a frictionless table. The outer ends of the springs are clamped while the springs are at their equilibrium lengths. Then a 2.0 N force applied to the box, parallel to the springs, compresses one spring by 3.0 cm while stretching the other by the same amount. What is the spring constant of the springs?

58. || A spring of equilibrium length L_1 and spring constant k_1 hangs from the ceiling. Mass m_1 is suspended from its lower end. Then a second spring, with equilibrium length L_2 and spring constant k_2 , is hung from the bottom of m_1 . Mass m_2 is suspended from this second spring. How far is m_2 below the ceiling?

59. || A horizontal spring with spring constant 250 N/m is compressed by 12 cm and then used to launch a 250 g box across the floor. The coefficient of kinetic friction between the box and the floor is 0.23. What is the box's launch speed?

60. | A 90 kg firefighter needs to climb the stairs of a 20-m-tall building while carrying a 40 kg backpack filled with gear. How much power does he need to reach the top in 55 s?

61. || A hydroelectric power plant uses spinning turbines to transform the kinetic energy of moving water into electric energy with 80% efficiency. That is, 80% of the kinetic energy becomes electric energy. A small hydroelectric plant at the base of a dam generates 50 MW of electric power when the falling water has a speed of 18 m/s. What is the water flow rate—kilograms of water per second—through the turbines?

62. || When you ride a bicycle at constant speed, nearly all the energy you expend goes into the work you do against the drag force of the air. Model a cyclist as having cross-section area 0.45 m^2 and, because the human body is not aerodynamically shaped, a drag coefficient of 0.90.

- What is the cyclist's power output while riding at a steady 7.3 m/s (16 mph)?
- Metabolic power* is the rate at which your body "burns" fuel to power your activities. For many activities, your body is roughly 25% efficient at converting the chemical energy of food into mechanical energy. What is the cyclist's metabolic power while cycling at 7.3 m/s?
- The food calorie is equivalent to 4190 J. How many calories does the cyclist burn if he rides over level ground at 7.3 m/s for 1 h?

63. || A farmer uses a tractor to pull a 150 kg bale of hay up a 15° incline to the barn at a steady 5.0 km/h. The coefficient of kinetic friction between the bale and the ramp is 0.45. What is the tractor's power output?

64. || A Porsche 944 Turbo has a rated engine power of 217 hp. 30% of the power is lost in the drive train, and 70% reaches the wheels. The total mass of the car and driver is 1480 kg, and two-thirds of the weight is over the drive wheels.

a. What is the maximum acceleration of the Porsche on a concrete surface where $\mu_s = 1.00$?

Hint: What force pushes the car forward?

b. If the Porsche accelerates at a_{max} , what is its speed when it reaches maximum power output?

c. How long does it take the Porsche to reach the maximum power output?

65. || Astronomers using a 2.0-m-diameter telescope observe a distant supernova—an exploding star. The telescope's detector records $9.1 \times 10^{-11} \text{ J}$ of light energy during the first 10 s. It's known that this type of supernova has a visible-light power output of $5.0 \times 10^{37} \text{ W}$ for the first 10 s of the explosion. How distant is the supernova? Give your answer in *light years*, where one light year is the distance light travels in one year. The speed of light is $3.0 \times 10^8 \text{ m/s}$.

66. || Six dogs pull a two-person sled with a total mass of 220 kg. The coefficient of kinetic friction between the sled and the snow is 0.080. The sled accelerates at 0.75 m/s^2 until it reaches a cruising speed of 12 km/h. What is the team's (a) maximum power output during the acceleration phase and (b) power output during the cruising phase?

In Problems 67 through 69 you are given the equation(s) used to solve a problem. For each of these, you are to

- Write a realistic problem for which this is the correct equation(s).
- Draw a pictorial representation.
- Finish the solution of the problem.

$$67. \frac{1}{2}(2.0 \text{ kg})(4.0 \text{ m/s})^2 + 0 + (0.15)(2.0 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) = 0 + 0 + T(2.0 \text{ m})$$

$$68. F_{\text{push}} - (0.20)(30 \text{ kg})(9.8 \text{ m/s}^2) = 0 \\ 75 \text{ W} = F_{\text{push}} v$$

$$69. T - (1500 \text{ kg})(9.8 \text{ m/s}^2) = (1500 \text{ kg})(1.0 \text{ m/s}^2) \\ P = T(2.0 \text{ m/s})$$

Challenge Problems

70. || A 12 kg weather rocket generates a thrust of 200 N. The rocket, pointing upward, is clamped to the top of a vertical spring. The bottom of the spring, whose spring constant is 550 N/m, is anchored to the ground.

- Initially, before the engine is ignited, the rocket sits at rest on top of the spring. How much is the spring compressed?
- After the engine is ignited, what is the rocket's speed when the spring has stretched 40 cm?

71. || A gardener pushes a 12 kg lawnmower whose handle is tilted up 37° above horizontal. The lawnmower's coefficient of rolling friction is 0.15. How much power does the gardener have to supply to push the lawnmower at a constant speed of 1.2 m/s? Assume his push is parallel to the handle.

72. || A uniform solid bar with mass m and length L rotates with angular velocity ω about an axle at one end of the bar. What is the bar's kinetic energy?